COUPLED RADIATION BETWEEN CONCENTRICALLY PLACED WAVEGUIDE APPLICATORS: OPTIMIZATION OF THE DEPOSITED POWER DISTRIBUTION INSIDE A LOSSY MEDIUM

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ABSTRACT

A method is proposed for determining the optimal amplitude and phase excitations of a phased array hyperthermia system consisting of four waveguide applicators, in order to attain an improved Specific Absorption Rate (SAR) distribution inside and outside of malignant tissues. The method is based on a rigorous electromagnetic analysis which takes into account coupling phenomena between array elements. A penalty function technique, using the downhill simplex method, is applied to solve the optimization problem. Numerical simulations have been performed to check the effectiveness of the proposed method.

I. INTRODUCTION

Recently, phased array hyperthermia systems have received considerable attention [1]-[2], because they provide the possibility of improving the penetration depth of electromagnetic power inside the body and give an effective means of controling the electromagnetic energy deposited in tissues, by adjusting the amplitudes and phases of the array applicators. Different bioheat transfer models [3] have been used for determining the required SAR distribution inside tissues, in order to ensure that the tumor will be maximally heated while the surrounding healthy tissues will be kept below a certain safe level. Thus, by controling the SAR distribution, an improved temperature distribution can be achieved. Several methods have been proposed in the past for the optimization of either SAR [4] or temperature [5] distributions.

In this paper, an optimization technique is proposed in order to determine optimal amplitude and phase excitations for the waveguide applicators of a phased array hyperthermia system. Contrary to the optimization techniques previously published, in this work a detailed three - dimensional electromagnetic model of the heated area which takes into account coupling phenomena between system elements, is employed.

Fig. 1 depicts the assumed geometry of a phased array using four identical waveguide applicators positioned on the surface of a layered cylindrical lossy medium. The signals emitted from each applicator are assumed to be coherent and the whole system is defined in terms of two row vectors

$$\underline{p} = \left[p_1 \; p_2 \; p_3 \; p_4\right]^T \qquad \underline{\varphi} = \left[\varphi_1 \; \varphi_2 \; \varphi_3 \; \varphi_4\right]^T$$

where p_ℓ and φ_ℓ are ℓth array element amplitude and phase, respectively, and T denotes transposition. Given the state vectors $\underline{p}$ and $\underline{\varphi}$ of the array, let $\underline{E}(\underline{r})$ be the electric field predicted by the model at point $\underline{r}$ in tissue. Then the proposed optimization method is expressed by the following requirements:

(a) Minimization of the quadratic error function

$$\iiint_{Tumor}\left[|\underline{E}(\underline{r})|^2-(E_{des}(\underline{r}))^2\right]^2 d\underline{r} \text{ for tumor region} \qquad (1)$$

and

$$\text{(b) } |\underline{E}(\underline{r})|^2 \le (E_{des}(\underline{r}))^2 \text{ for healthy tissues} \qquad (2)$$

where $(E_{des}(\underline{r}))^2$ reflects the desired SAR distribution inside and outside the tumor.

II. ELECTROMAGNETIC MODELING

The phased array hyperthermia system examined in this paper consists of four identical rectangular waveguide applicators surrounding a three-layered cylindrical body model, the three layers corresponding to muscle, fat and skin and having relative permittivity ε_i and relative permeability μ_i, i=1,2,3, respectively. The free-space wavenumber is $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, where ε_0 and μ_0 are the free-space

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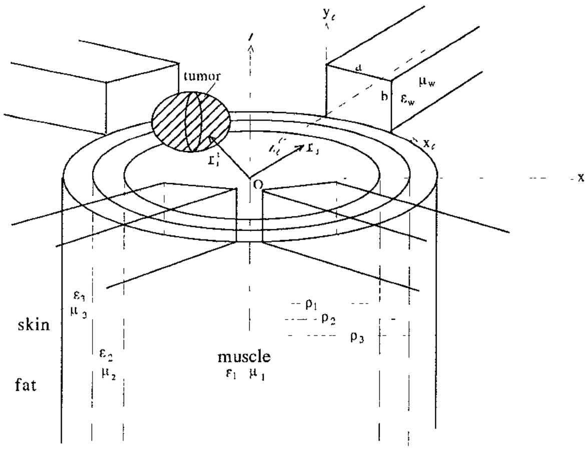

Fig. 1: Tissue geometry and the phased array hyperthermia system.

permittivity and permeability, respectively. The applicators are filled with a dielectric material of relative permittivity ε_w and relative permeability μ_w. An integral equation technique has been adopted in order to determine the electromagnetic field distribution inside the body. In the following analysis, an exp(+jωt) time dependence is assumed for the field quantities.

Cylindrical wave functions $\underline{M}^{(q)}_{m,k}(\underline{r},k_i)$ and $\underline{N}^{(q)}_{m,k}(\underline{r},k_i)$ q=1,2 [6] are used to express the solution of the wave equation inside the tissue

$$\underline{E}_i(\underline{r}) = \int_{-\infty}^{+\infty} dk \sum_{m=-\infty}^{m=+\infty} \begin{pmatrix} a_{im}\underline{M}^{(1)}_{m,k}(\underline{r},k_i) + b_{im}\underline{N}^{(1)}_{m,k}(\underline{r},k_i) \\ +a'_{im}\underline{M}^{(2)}_{m,k}(\underline{r},k_i) + b'_{im}\underline{N}^{(2)}_{m,k}(\underline{r},k_i) \end{pmatrix} \quad (3)$$

where i = 1,2,3 corresponds to the three regions, $k_i = k_0\sqrt{\varepsilon_i\mu_i}$ and $a_{im}, b_{im}, a'_{im}, b'_{im}$ are uknown coefficients to be determined.

Then, the fields inside each waveguide are described as the superposition of the incident TE_{10} mode and an infinite number of all the reflected normal modes. The transverse electric field inside the ℓth waveguide applicator (ℓ=1,...,4) can be written, with respect to the local cartesian coordinates system x_ℓ, y_ℓ, z_ℓ (Fig. 1), as follows [6]

$$\underline{E}^{(\ell)}_{w,t}(\underline{r}) = p_\ell e^{j\varphi_\ell} \underline{e}^{TE}_{1,t}(x_\ell,y_\ell) \frac{j\omega\mu_0\mu_w}{u_1} e^{-j\gamma_1 z_\ell} + \sum_{n=1}^{\infty} \begin{pmatrix} A^{(\ell)}_n \underline{e}^{TE}_{n,t}(x_\ell,y_\ell) \frac{j\omega\mu_0\mu_w}{u_n} e^{j\gamma_n z_\ell} \\ +B^{(\ell)}_n \underline{e}^{TM}_{n,t}(x_\ell,y_\ell)\left(-\frac{j\lambda_n}{v_n}\right) e^{j\lambda_n z_\ell} \end{pmatrix} \quad (4)$$

where the subscript t is used to denote the transverse field components, $\underline{e}^{TE}_{n,t}$, $\underline{e}^{TM}_{n,t}$ are TE and TM modal fields [6] and γ_n, λ_n the corresponding propagation constants of these fields.

By satisfying the continuity of the tangential electric and magnetic field components on the $\rho=\rho_1$ and $\rho=\rho_2$ interfaces and on the $\rho=\rho_3$ contact surface between cylindrical lossy model and radiating apertures, the following system of four coupled integral equations is obtained in terms of the unknown transverse electric field components $\underline{E}_a(\varphi',z')$ on the waveguide apertures

$$\sum_{i=1}^{4} \iint_{S_i} \rho_3 d\varphi' dz' \underline{\overline{K}}_{\ell i}(\varphi,z/\varphi',z') \underline{E}_a(\varphi',z') = 2p_\ell e^{j\varphi_\ell} \underline{h}^{TE}_{1,t}(\varphi,z)\left(\frac{j\gamma_1}{u_1}\right) \quad \ell = 1,\ldots,4 \quad (5)$$

where $\underline{h}^{TE}_{1,t}$ is the incident TE_{10} mode transverse magnetic field on the aperture of the ℓth waveguide applicator, and $\underline{\overline{K}}_{\ell i}(\varphi,z/\varphi',z')$, $\ell = 1,\ldots,4$, $i = 1,..,4$ are kernel matrices.

In order to determine the electric field on the waveguide apertures, the system of integral equations (5) is solved. To this end a Galerkin's technique is adopted, by expanding the transverse electric field $\underline{E}^{(i)}_a$ on each aperture into wavequide normal modes

$$\underline{E}^{(i)}_a = \sum_{n=1}^{\infty}\left(g^{(i)}_n \underline{e}^{TE}_{n,t} + p^{(i)}_n \underline{e}^{TM}_{n,t}\right) \quad i = 1,\ldots,4 \quad (6)$$

Once the transverse electric fields on the apertures are determined, the electric field at any point within the body can be computed by using the expression (3).

III. FORMULATION AND SOLUTION OF THE OPTIMIZATION PROBLEM

In reference to the geometry of Fig. 1, let p_ℓ and φ_ℓ, ℓ=1,..,4 be arbitrary values of amplitude and phase for the ℓth applicator. The optimization problem may be formulated as the minimization, with respect to p_ℓ and φ_ℓ of the error function (1) subject to the inequality constraints (2), where $\underline{E}(\underline{r})$ in (1) and (2) is computed by (3), as it has been explained in Section II.

In order to proceed with the optimization procedure the quadratic error function defined in (1) for the tumor volume and the condition (2) for the healthy tissue have been discretized. The integral in (1) has been replaced by the function

$$f(\underline{p},\underline{\varphi}) = \sum_{i=1}^{m} \left[|\underline{E}(\underline{p},\underline{\varphi};\underline{r}_i^t)|^2 - (E_{des}(\underline{r}_i^t))^2 \right]^2 \qquad (7)$$

and the constraints (2) have been replaced by the finite set

$$|\underline{E}(\underline{p},\underline{\varphi};\underline{r}_j)|^2 \leq (E_{des}(\underline{r}_j))^2 \quad j = 1,2,\ldots,n \qquad (8)$$

The optimization problem is then stated as follows

$$\min_{\underline{p},\underline{\varphi}} \left\{ f(\underline{p},\underline{\varphi}) : \; |\underline{E}(\underline{p},\underline{\varphi};\underline{r}_j)|^2 \leq (E_{des}(\underline{r}_j))^2 \;\; j = 1,2,\ldots,n \right\} \qquad (9)$$

The following observations lead to a simplification of problem (9). The solution of the electromagnetic problem in Section II involves phase differences rather than phase values. Therefore, one of the phases, say φ_4, may be taken as reference ($\varphi_4=0$), leaving only 3 independent phase variables for problem (9). In addition, because the method is mainly aimed to optimize relative SAR distributions, rather than absolute SAR values, one of the amplitudes, say p_4, may be taken as a reference in computing the electromagnetic field, by using the analysis presented in Section II. Thus, the normalized SAR distribution $\propto |\tilde{\underline{E}}(\underline{r})|^2 / p_4^2$ must be optimized with respect to the following 6 independent optimization variables

$$\underline{\tilde{p}} = \left[p_1 \; p_2 \; p_3 \right]^T \qquad \underline{\tilde{\varphi}} = \left[\varphi_1 \; \varphi_2 \; \varphi_3 \right]^T$$

The value of the reference amplitude p_4 is taken to be such that the calculated maximum of $|\underline{E}(\underline{r})|^2 / p_4^2$ over the m+n discretization points is equal to 1.

The penalty function approach [7] is used for the solution of (9), leading to a sequence of unconstraint minimization problems of the form

$$\min_{\underline{\tilde{p}},\underline{\tilde{\varphi}}} \left\{ q(\underline{\tilde{p}},\underline{\tilde{\varphi}};\mu) \right\} \qquad (10)$$

where

$$q(\underline{\tilde{p}},\underline{\tilde{\varphi}};\mu) = f(\underline{\tilde{p}},\underline{\tilde{\varphi}})$$
$$+\mu \sum_{j=1}^{n} \left[\max\left\{ 0, |\underline{E}(\underline{\tilde{p}},\underline{\tilde{\varphi}};\underline{r}_j)|^2 - (E_{des}(\underline{r}_j))^2 \right\} \right]^2 \qquad (11)$$

The second term in expression (11) represents a penalty associated with excessive SAR outside of the tumor region and μ is a positive penalty parameter. By increasing the parameter μ, a heavier penalty is placed on violation of the constraints outside the tumor. A general optimization procedure proposed by Nelder and Mead [8], the downhill simplex method, has been employed in order to solve problem (10).

IV. RESULTS AND DISCUSSION

The method developed here has been applied to study the performance of a four element phased array system, operating at 433 MHz. The applicators are water filled, have an aperture size of 5.8×2.9 cm^2 and are symmetrically placed at the periphery of a three-layered cylindrical body model of circular cross section, 10 cm in diameter. A spherical tumor with a 2 cm diameter is assumed and the optimization algorithm has been applied for two different tumor locations inside the body. In both

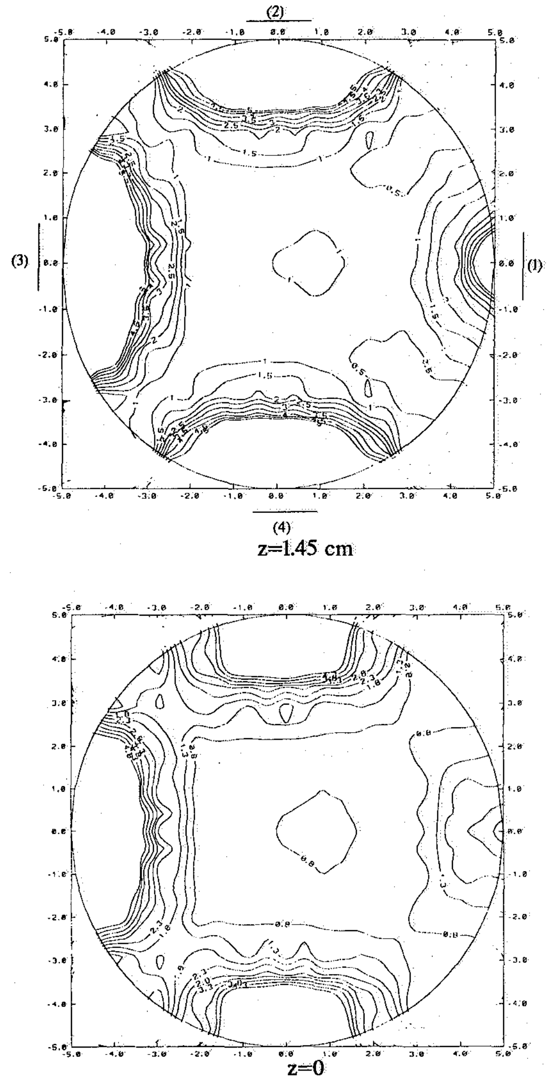

Fig. 2: SAR distribution at z=1.45 cm and z=0 in a cylindrical body, irradiated by four applicators of 5.8×2.9 cm^2 aperture size, symmetrically placed at the periphery, after optimization of the applicators excitation for a tumor 2 cm in diameter centered at point $1\hat{x} + 1.45\hat{z}$ (cm).

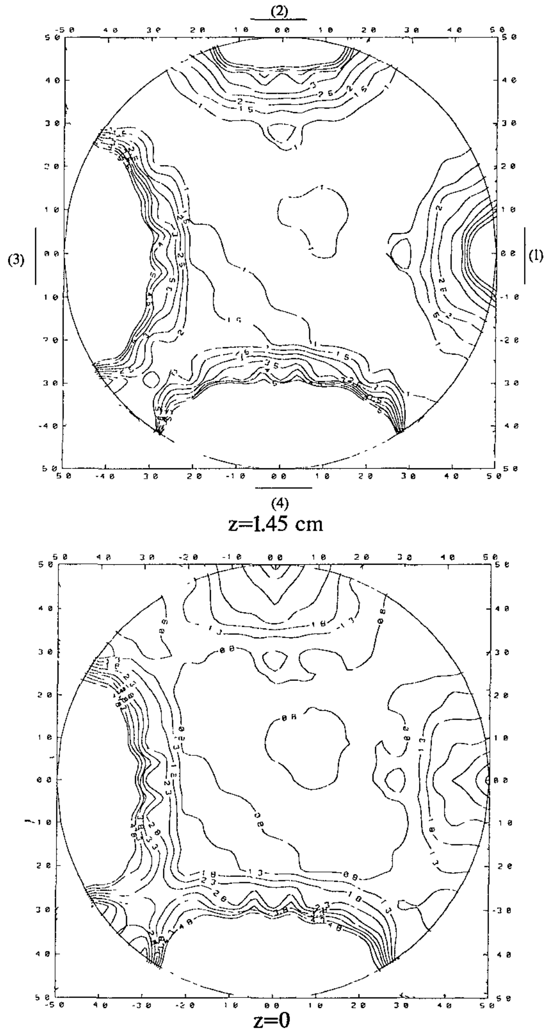

Fig. 3: SAR distribution at z=1.45 cm and z=0 for the same setting as in Fig. 2, after optimization of the applicators excitations for a tumor centered at point $1\hat{x} + 1\hat{y} + 1.45\hat{z}$ (cm).

cases the penalty parameter is taken to be μ=1 and the sets of discretization points consist of nine points: five points inside the tumor area (one point at the center and four points on the boundary between tumor and healthy tissue) and four points inside the healthy tissue, lying at 1 cm distance from the tumor boundary. The solution of problem (10) for a tumor centered at point $1\hat{x} + 1.45\hat{z}$ (cm) corresponds to the excitation p_1=0.56, p_2=1.0, p_3=1.42 and φ_1=-32°, φ_2=0°, φ_3=48.2°, and the calculated SAR distributions are shown in Fig. 2 at midplane and lower plane of the radiating apertures. The SAR distribution shown in Fig. 3 corresponds to the excitation p_1=p_2=0.4, p_3=1.0 and φ_1=φ_2=-88.8°, φ_3=0° obtained by applying the optimization procedure for a tumor centered at point $1\hat{x} + 1\hat{y} + 1.45\hat{z}$ (cm).

Examination of the SAR distributions, presented in Figs. 2 and 3, shows that increased SAR values are achieved in the whole tumor volume for both tumor lacations. Since the tumor tends to accumulate more heat than the normal tissue because of sluggish blood flow [4], a hot zone in the tumor region will be built up, while the surrounding healthy tissue is not exposed to excessive heating. Superficial cooling can be used in order to avoid the hot spot formulation in front of the apertures centers due to high SAR values observed at these areas.

V. CONCLUSION

A method for the optimization of the excitation of a phased array hyperthermia system has been presented. Accurate three - dimensional electromagnetic modeling has been used in order to predict the SAR distribution inside a layered cylindrical lossy model. The total squared error between desired and model predicted SAR distributions inside tumor is minimized, subject to an upper bound for SAR distribution outside the tumor. Numerical results for a four element system indicate that satisfactory SAR distributions can be obtained for different tumor locations inside the heated body.

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